**Mathematical Induction 3**

1. Prove that :

Let be the proposition: .

We like to use the Principle of Mathematical Induction to prove that is true .

For . is true.

Assume is true for some , that is,

For , , by (\*)

 is true.

 By the Principle of Mathematical Induction, is true .

2. Prove
  by mathematical induction.

The magic is to change the right hand side of the proposition to:

For , L.H.S. = , R.H.S. =

Assume is true for some , that is

For ,

 , by (1).

 is true.

 By the principle of mathematical induction, is true for all .

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 Why: Let

 It is not difficult to prove .

 Hence

3. Prove by any method.

 **Method 1 Mathematical Induction**

 Let

 For , , is true.

 Assume is true for some that is

 For ,

 , by (1)

 is true.

 By the principle of mathematical induction, is true for all

 **Method 2 Difference method (1)**

 Let

 Taking summation

 **Method 3 Difference method (2)**

 Consider

4.

 Let

 For , , is true.

 Assume is true for some that is

 For ,

 (Integration by parts)

 Solve for and by (1) we have

 is true.

 By the Principle of mathematical induction, is true for all

5. Prove:

 .

 Redefine the proposition into equivalent proposition:

 Let (a)

 (b)

 For (a) LHS = 1= = RHS

 (b) LHS == 2 = = RHS

 is true.

 Assume is true for some that is

 (a)

 (b)

 For

 (a) , by (1)

 (b)

 , by (2)

 is true.

 By the Principle of mathematical induction, is true for all **.**

6. Prove is divisible by for all non-negative integer value of .

 Let where .

 For

 Assume is true for some

 We have

 For ,

 , by (1)

 is true.

 By the Principle of mathematical induction, is true for all **.**

7. Prove, by Mathematical Induction, that is divisible by
 for all natural numbers , where

 Let is divisible by

 That is, is divisible by

 is divisible by

 is divisible by

 is divisible by

 …

 is divisible by

 For , is obviously divisible by .

 Assume is true for some , that is

 That is, is divisible by

 is divisible by

 is divisible by

 is divisible by

 …

 is divisible by

 is divisible by

 For ,

 (1) is divisible by is true

 (2) is divisible by since
 by is divisible by and is also divisible by

 (3)

 is divisible by , by

 is divisible by (just proved in (2))

 is divisible by

 Therefore is divisible by

 (4)
 is divisible by by

 is divisible by (just proved in (3))

 is divisible by

 Therefore is divisible by

 (5) …. Up to

 Continue in this way, by writing

 is divisible by since

 is divisible by by

 is divisible by

 is divisible by

 Therefore is divisible by

 is true.

 By the Principle of mathematical induction, is true **.**

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