**Mathematical Induction 3**

1. Prove that :

Let be the proposition: .

We like to use the Principle of Mathematical Induction to prove that is true .

For . is true.

Assume is true for some , that is,

For , , by (\*)

is true.

By the Principle of Mathematical Induction, is true .

2. Prove   
  by mathematical induction.

The magic is to change the right hand side of the proposition to:

For , L.H.S. = , R.H.S. =

Assume is true for some , that is

For ,

, by (1).

is true.

By the principle of mathematical induction, is true for all .

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Why: Let

It is not difficult to prove .

Hence

3. Prove by any method.

**Method 1 Mathematical Induction**

Let

For , , is true.

Assume is true for some that is

For ,

, by (1)

is true.

By the principle of mathematical induction, is true for all

**Method 2 Difference method (1)**

Let

Taking summation

**Method 3 Difference method (2)**

Consider

4.

Let

For , , is true.

Assume is true for some that is

For ,

(Integration by parts)

Solve for and by (1) we have

is true.

By the Principle of mathematical induction, is true for all

5. Prove:

.

Redefine the proposition into equivalent proposition:

Let (a)

(b)

For (a) LHS = 1= = RHS

(b) LHS == 2 = = RHS

is true.

Assume is true for some that is

(a)

(b)

For

(a) , by (1)

(b)

, by (2)

is true.

By the Principle of mathematical induction, is true for all **.**

6. Prove is divisible by for all non-negative integer value of .

Let where .

For

Assume is true for some

We have

For ,

, by (1)

is true.

By the Principle of mathematical induction, is true for all **.**

7. Prove, by Mathematical Induction, that is divisible by   
 for all natural numbers , where

Let is divisible by

That is, is divisible by

is divisible by

is divisible by

is divisible by

…

is divisible by

For , is obviously divisible by .

Assume is true for some , that is

That is, is divisible by

is divisible by

is divisible by

is divisible by

…

is divisible by

is divisible by

For ,

(1) is divisible by is true

(2) is divisible by since   
 by is divisible by and is also divisible by

(3)

is divisible by , by

is divisible by (just proved in (2))

is divisible by

Therefore is divisible by

(4)   
 is divisible by by

is divisible by (just proved in (3))

is divisible by

Therefore is divisible by

(5) …. Up to

Continue in this way, by writing

is divisible by since

is divisible by by

is divisible by

is divisible by

Therefore is divisible by

is true.

By the Principle of mathematical induction, is true **.**

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